SSVI a la Bergomi

Stefano De Marco[1], Claude Martini[2]

[1] Ecole Polytechnique
[2] Zeliade Systems

Jim Gatheral 60th birthday conference

Foreword To Jim

A nice pipeline:

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Jim's → *Zeliade (ZQF, Model Validation)* → *Banks, HFs, CCPs*

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- SVI
- SSVI

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- ..and more to come!

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So, Jim, on behalf of Zeliade I say: thank you!

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1st ingredient: (e)SSVI

Gatheral SVI

Formula for the implied total variance *at a given maturity T*:

$$v(k) = a + b(\rho(k - m) + \sqrt{(k - m)^2 + \sigma^2})$$

where:

v = implied vol$^2 T$.

k is the log forward moneyness.

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Fits super well (the best 5 parameters model around?).

Gatheral-Jacquier *Surface* SVI

Formula for the implied total variance *for the whole surface*:

$$w(k,\theta_t) = \frac{\theta_t}{2}(1 + \rho\varphi(\theta_t)k + \sqrt{(\varphi(\theta_t)k + \rho)^2 + \bar{\rho}^2})$$

where:

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(Historically, stems out of SVI. SSVI *slices* are a subfamily of SVI).

No arbitrage in SSVI

Proposition (GJ, SSVI paper, Theorems 4.1 and 4.2)

There is no calendar spread and no butterfly arbitrage if

$$\partial_t \theta_t \geq 0 \tag{2.1}$$

$$0 \leq \partial_\theta(\theta\varphi(\theta)) \leq \frac{1}{\rho^2}(1+\bar{\rho})\varphi(\theta), \ \forall \theta > 0 \tag{2.2}$$

$$\theta\varphi(\theta) \leq \min\left(\frac{4}{1+|\rho|}, 2\sqrt{\frac{\theta}{1+|\rho|}}\right), \ \forall \theta > 0 \tag{2.3}$$

where $\bar{\rho} = \sqrt{1-\rho^2}$.

Condition 2.3 implies that $\lim_{\theta\to 0} \theta\varphi(\theta) = 0$.

SSVI in practice

Usage: implied vol smoother, risk models

Widely used on Equity (indexes, stocks), works very well

Also on some FI and FX markets

Easy to implement (calibration easier than SVI)

e(xtended) SSVI

(joint work with Sebas Hendriks)

Idea: allows for time (θ) dependent correlation ρ in SSVI.

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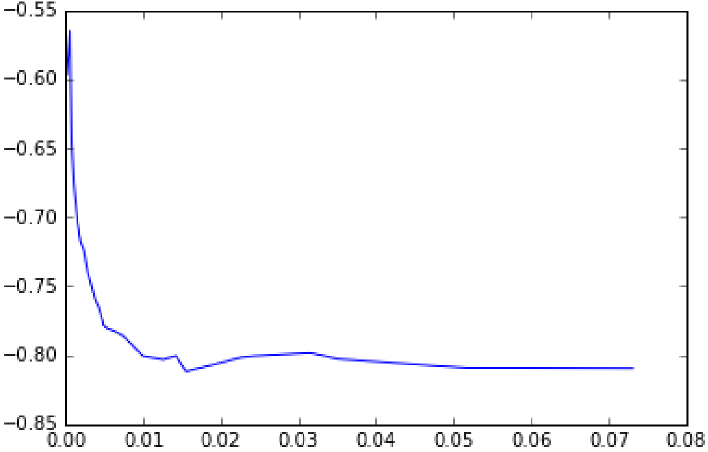
Idea: allows for time (θ) dependent correlation ρ in SSVI. Motivation: correlation in the calibration of a *joint slice SSVI* model:



e(xtended) SSVI

eSSVI *slices* are SSVI slices: same no-butterfly arbitrage conditions.

Question: investigate calendar-spread arbitrage.

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Question: investigate calendar-spread arbitrage.

Starting point: look at 2 SSVI slices with different correlations ρ_1, ρ_2.

$$
\begin{aligned}
w_1 &= \frac{\theta_1}{2}\left(1 + \rho_1 \varphi_1 k + \sqrt{\varphi_1^2 k^2 + 2\rho_1\varphi_1 k + 1}\right) \\
w_2 &= \frac{\theta_2}{2}\left(1 + \rho_2 \varphi_2 k + \sqrt{\varphi_2^2 k^2 + 2\rho_2\varphi_2 k + 1}\right)
\end{aligned}
\tag{2.4}
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[Haute Couture on *parametric quadratic polynomials* here]

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[Haute Couture on *parametric quadratic polynomials* here]

Proposition (Sufficient conditions for no crossing)

The 2 smiles don't cross if

$$
\theta_2 \geq \theta_1 \text{ and } \varphi_2 \leq \varphi_1
$$

$$
\frac{\theta_2\varphi_2}{\theta_1\varphi_1} \geq \max\left(\frac{1+\rho_1}{1+\rho_2}, \frac{1-\rho_1}{1-\rho_2}\right)
$$

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Proposition

Let

$$\gamma := \frac{1}{\varphi}\frac{d(\theta\varphi)}{d\theta}, \delta := \theta\frac{d(\rho)}{d\theta}$$

Then there is no calendar spread arbitrage in eSSVI iff $\partial_t\theta_t \geq 0$ *and*

$$-\gamma \leq \delta + \rho\gamma \leq \gamma$$

and either:

1. $\gamma \leq 1$
2. $-\sqrt{2\gamma - 1} \leq \delta + \rho\gamma \leq \sqrt{2\gamma - 1}$

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Can be proven rigorously directly, investigating $\partial_\theta w$.

Representation formula for $\rho(\theta)$

If we restrict to the case where $0 \leq \gamma \leq 1$, we can get all possible ρ satisfying $-\gamma \leq \delta + \rho\gamma \leq \gamma$ by solving the ODE $\delta + \rho\gamma = \gamma u$ where u is any function with values in $[-1, 1]$.

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Proposition

Assume $0 \leq \gamma \leq 1$ *Then there is no calendar spread arbitrage in eSSVI iff*

$$\rho(\theta) = \frac{1}{\theta\varphi(\theta)} \int_0^\theta u(\tau) d(\tau\varphi(\tau)) \tag{2.5}$$

for some $u \to [-1, 1]$

2nd ingredient: Chriss-Morokoff-Gatheral-Fukasawa formula

VIX reminder

For a continuous model:

$$\lim E[\sum \log \frac{S_{(k+1)h}}{S_{kh}}^2] = E[-2\log(\frac{S_T}{S_0})]$$

and one has always the replication formula for the log contract:

$$E[-2\log(\frac{S_T}{F_T})] = 2\int_0^{F_T} \frac{P(K,T)}{K^2}dK + 2\int_{F_T}^{\infty} \frac{C(K,T)}{K^2}dK$$

where we assume that there is no interest rate. Here $C(K,T)$ (resp. $P(K,T)$) is the price of a Call (resp. Put) with strike K and time to maturity T. F_T is the Forward at maturity T

VIX: synthetic index with a discrete version of this formula (and fixed 30 days time to maturity)

Notation: $VIX^2(T) = E[-2\log(\frac{S_T}{F_T})]/T$

Chriss-Morokoff-Gatheral-Fukasawa formula

In Jim's *Practitioner* book, the following formula is obtained:

$$E[-2\log(\frac{S_T}{F_T})] = \int \sigma^2(g_2(z))\frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}dz \tag{3.6}$$

(we drop the T dependence in the RHS)

where g_2 is the inverse function of the transformation $k \to d_2(k, \sigma(k))$ where $d_2(k, \sigma) = -\frac{k}{\sigma} - \frac{\sigma}{2}$.

Fukasawa (2010) proved that under no butterfly arbitrage conditions $d_2(k., \sigma(.))$ is indeed invertible and proved rigorously 3.6.

General shape of $\sigma(g_2)$

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Lemma (Fukasawa)

The inequality $2g_2(z) \leq z^2$ *holds for all* $z \in \mathbb{R}$. *There exists a unique* $z^* > 0$ *such that* $2g_2(z^*) = (z^*)^2$. *Moreover, we have* $\sigma(g_2(z)) = z + \sqrt{z^2 - 2g_2(z)}$ *below* z^* *and* $\sigma(g_2(z)) = z - \sqrt{z^2 - 2g_2(z)}$ *above* z^*. *In particular,* $\sigma(g_2(z^*)) = z^*$.

$\sigma(g_2)$ in SSVI

SSVI:

$$\sigma^2(g_2(z)) = \frac{\theta}{2}(1 + \rho\varphi g_2 + \sqrt{(\varphi g_2 + \rho)^2 + \bar{\rho}^2})$$

so $\theta(1 + \rho\varphi g_2 + \sqrt{(\varphi g_2 + \rho)^2 + \bar{\rho}^2}) = 4(z^2 - g_2 \pm z\sqrt{z^2 - 2g_2})$

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[EASY COMPUTATIONS HERE]

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[EASY COMPUTATIONS HERE]
Setting $v_2 = \sigma(g_2(z))$ we get the quadratic equation:

$$\theta^2(1 - \rho^2)\varphi^2 \frac{(2z - v_2)^2}{4} = 4[v_2^2 - \theta(1 + \rho\varphi \frac{v_2(2z - v_2)}{2})] \qquad (3.7)$$

Close formula for $\sigma(g_2)$ and the VIX in (e)SSVI

Let $u := \theta\varphi(\theta)$ and set:

$$a = 1 + \frac{\rho u}{2} - \frac{\bar{\rho}^2 u^2}{16}$$
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Proof.

$\sigma(g_2(z))^2 = \frac{(b^2+u^2)z^2+4a\theta-2bz\sqrt{u^2z^2+4a\theta}}{4a^2}$, integrate in z wrt Gauss kernel.

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Some properties

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Assume $\rho \leq 0$ and no calendar-spread arbitrage. Then:

1. $\theta \to V(\theta)$ is non-decreasing.
2. $V(\theta) \geq \theta$

Conclusion: (e)SSVI a la Bergomi

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Why *a la Bergomi?*
$T\ VIX^2(T) = \int_0^T \xi_0(t)dt$ where ξ_0 is the intial Forward Variance curve.
Key input of Bergomi approach.

From V to θ

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Same *principle* for $\varphi(\theta) = \eta\theta^{-\lambda}$ with $\lambda \neq 1/2$.

$\theta(V)$ formulas

Proposition (ATM implied total variance, uncorrelated sqrt SSVI)

Assume $\varphi(\theta) = \frac{\eta}{\sqrt{\theta}}$ *and* $\rho = 0$. *Then:*

$$\theta = \frac{8(1 - \sqrt{1 + \frac{\eta^2}{2} + \frac{\eta^4}{8}(V + \frac{1}{2})}) + \eta^2(V + 2)}{\eta^2(1 + \frac{\eta^2(V-4)}{16})}$$

Proposition (ATM implied total variance, uncorrelated sqrt SSVI, small parameter expansion)

Assume $\varphi(\theta) = \frac{\eta}{\sqrt{\theta}}$ *and* $\rho = 0$. *Then at first order in* η^2*:*

$$\theta = V(1 - \frac{\eta^2(V + 4)}{16})$$

$\theta(V)$ formulas,ctd

Proposition (Short term ATM implied total variance, sqrt SSVI)

Assume $\varphi(\theta) = \frac{\eta}{\sqrt{\theta}}$. *Then for small* θ*:*

$$\theta = \frac{V}{\left(\frac{(1+\rho^2)}{4}\eta^2 + 1\right)}\left[1 + \rho\eta\sqrt{V}\frac{\left(\frac{(3+\rho^2)}{8}\eta^2 + \frac{1}{2}\right)}{\left(\frac{(1+\rho^2)}{4}\eta^2 + 1\right)^{\frac{3}{2}}} - \eta^2 V \frac{\frac{(3\rho^2+1)}{16} + \eta^2\frac{3(\rho^4+6\rho^2+1)}{64}}{\left(\frac{(1+\rho^2)}{4}\eta^2 + 1\right)^2} + o(V)\right]$$

SSVI a la Bergomi

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with one of these formulas. Same parameters as Bergomi type models.

Vol and correlation are disantangled.

Hurst exponent from short term skew in Rough Bergomi models

Identification of the short term skew:

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Identification of the short term skew:

(SSVI)

$$\sqrt{T}\partial_k \sigma_{BS}(k=0) \approx \rho/2\sqrt{\theta}\varphi(\theta) \propto \rho T^{1/2-\lambda}$$

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(Rough Bergomi: Fukasawa, 2015)

$$\propto \rho T^{H}$$

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Temptative rough (e)SSVI

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Temptative rough (e)SSVI

Thank you for your attention !

Thanks and joyeux anniversaire Jim !!!